Chiral kinetic theory and Berry phase

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This talk bases on:

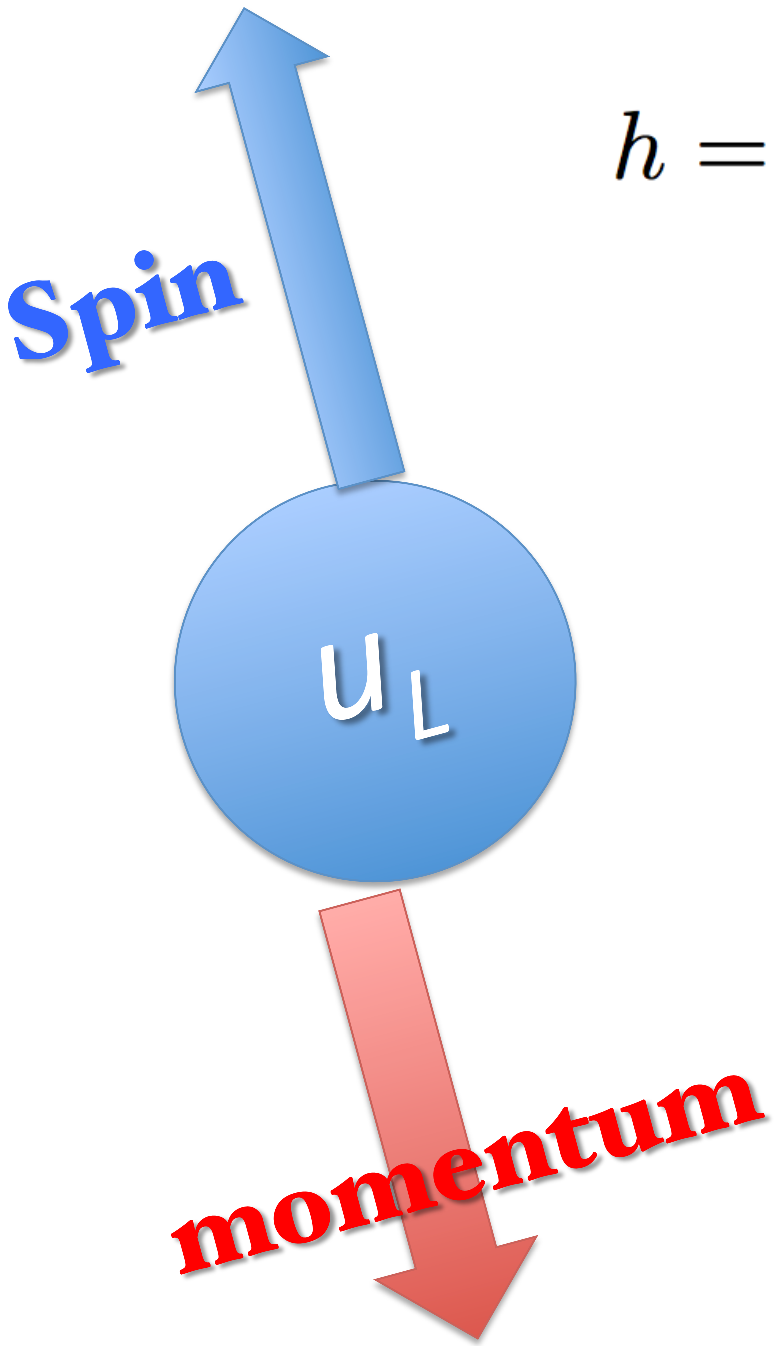
- Jian-hua Gao, Zuo-Tang Liang, SP, Qun Wang, Xin-Nian Wang, Phys. Rev. Lett. 109 (2012) 232301
- Jiunn-Wei Chen, SP, Qun Wang, Xin-nian Wang, Phys.Rev.Lett. 110 (2013) 262301

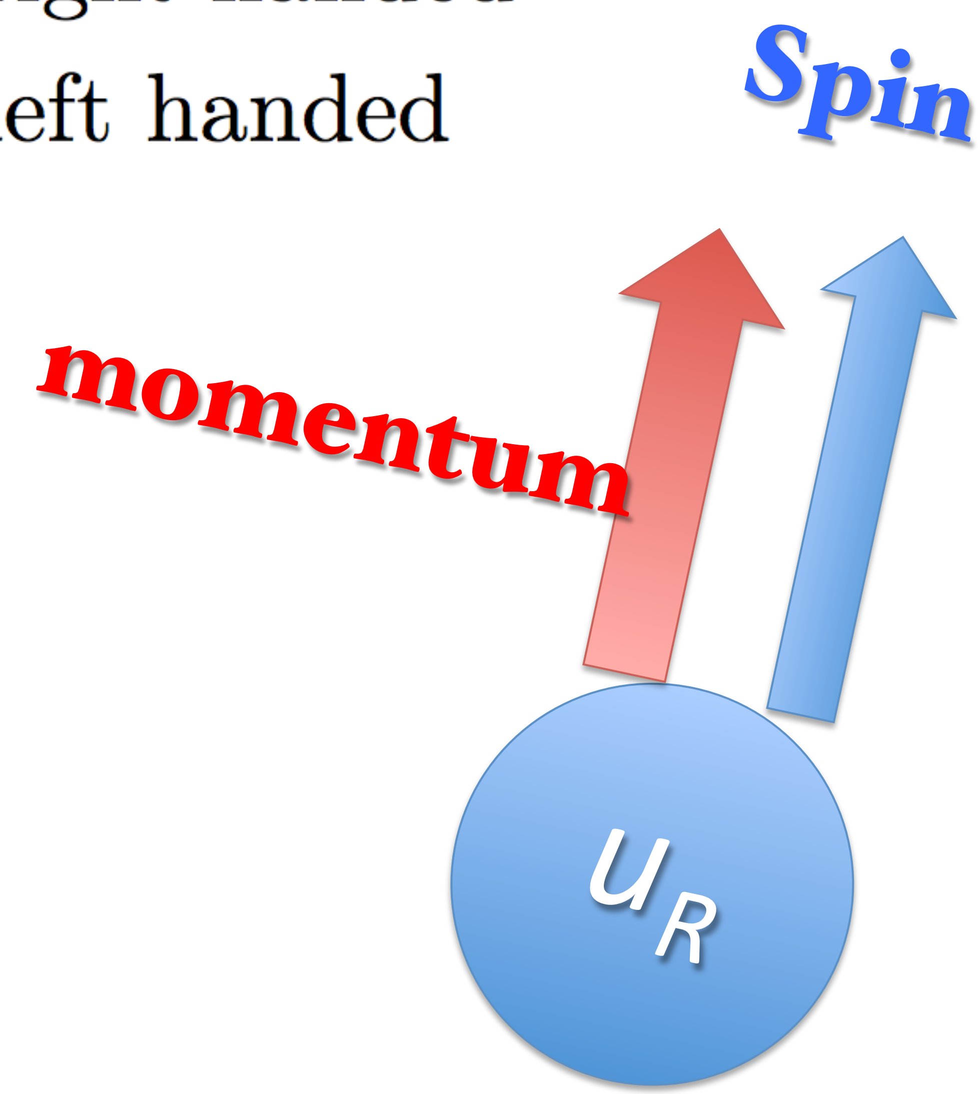
Outline

- Chiral magnetic and vortical effects
- Quantum kinetic theory
- Chiral kinetic theory with Berry phase
- Summary

Chirality of massless fermions

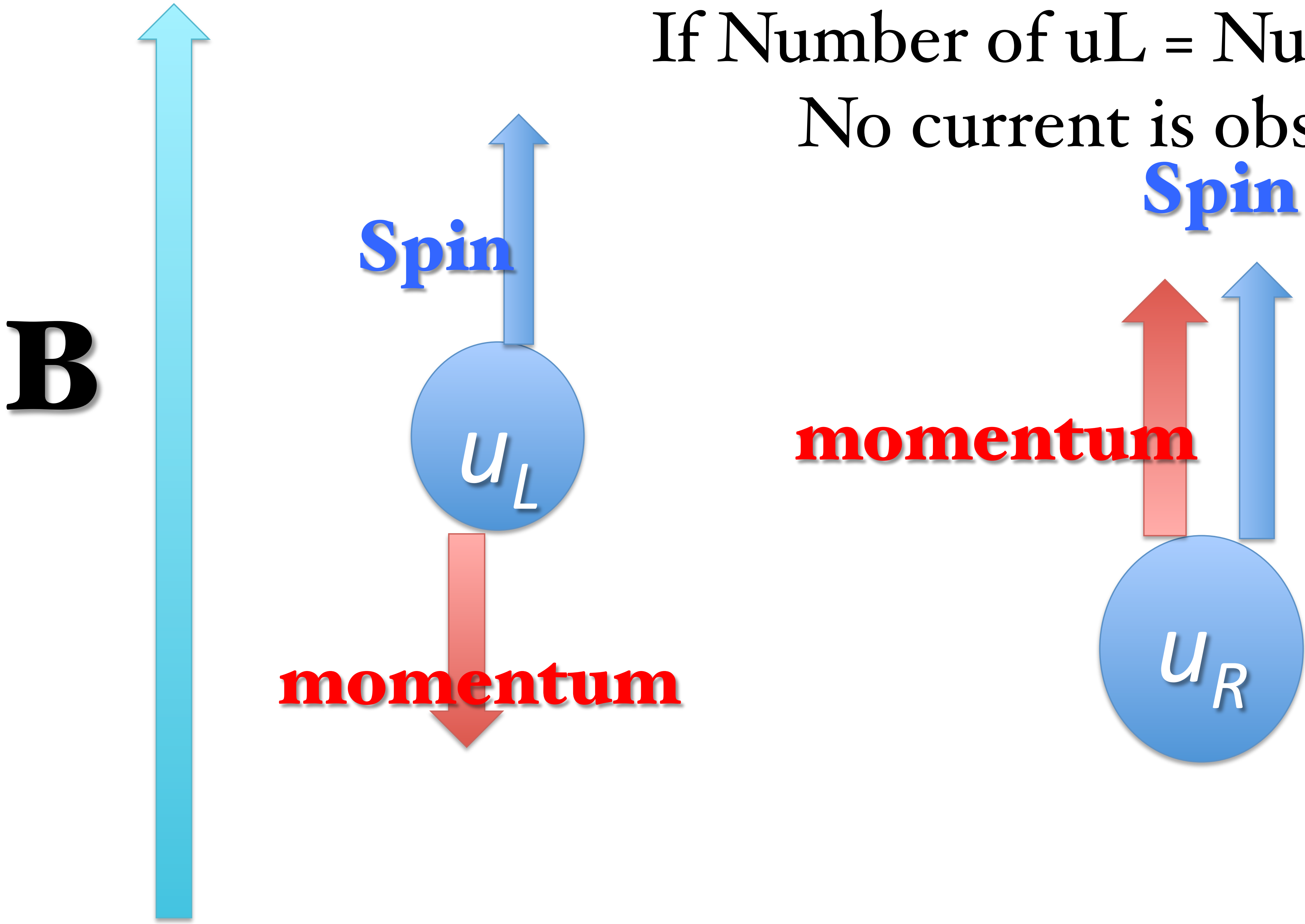
$$h = \frac{\sigma \cdot p}{|\mathbf{p}|} = \begin{cases} +1, & \text{right handed} \\ -1, & \text{left handed} \end{cases}$$





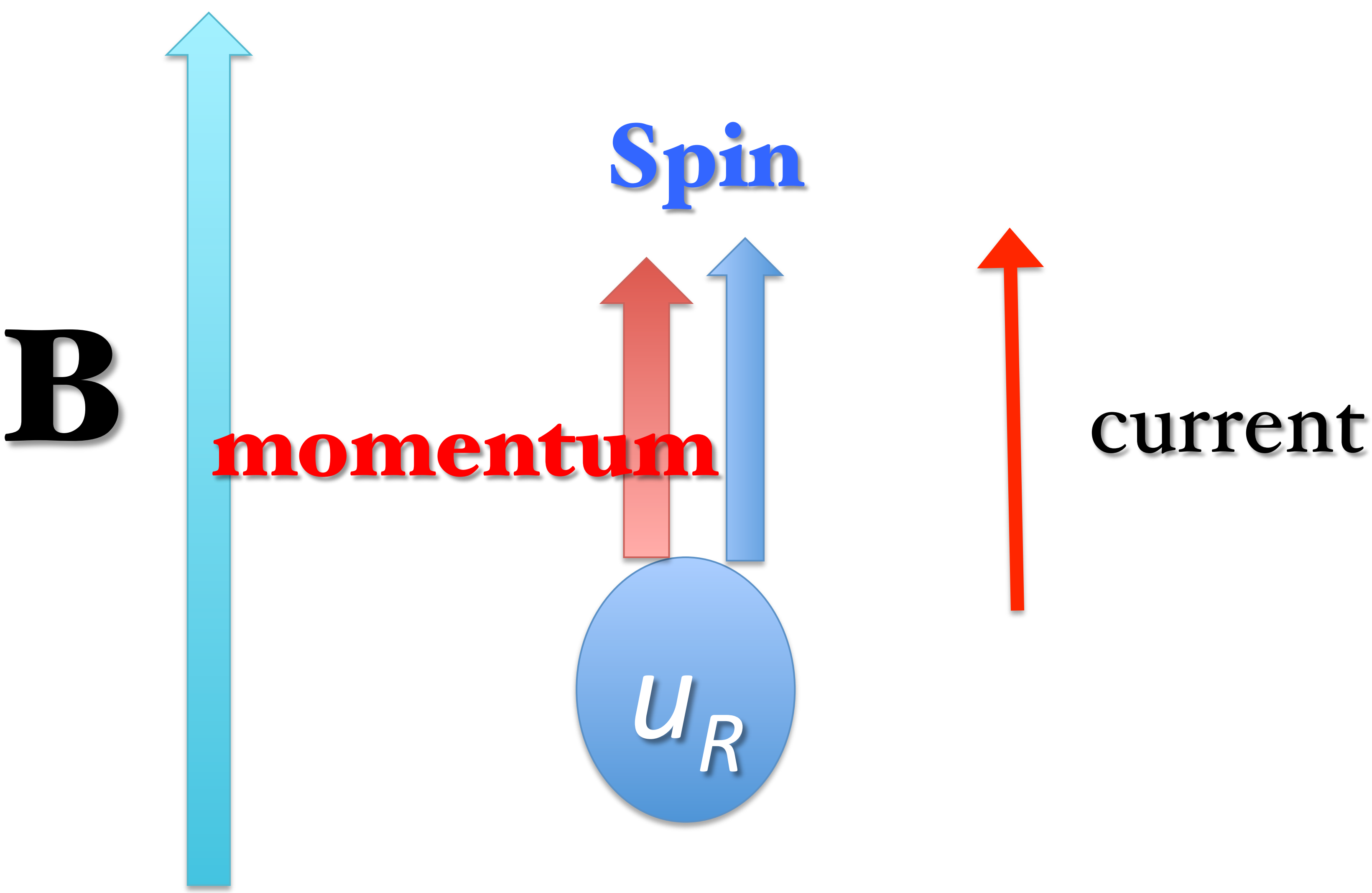
Chirality

If Number of uL = Number of uR
No current is observed.

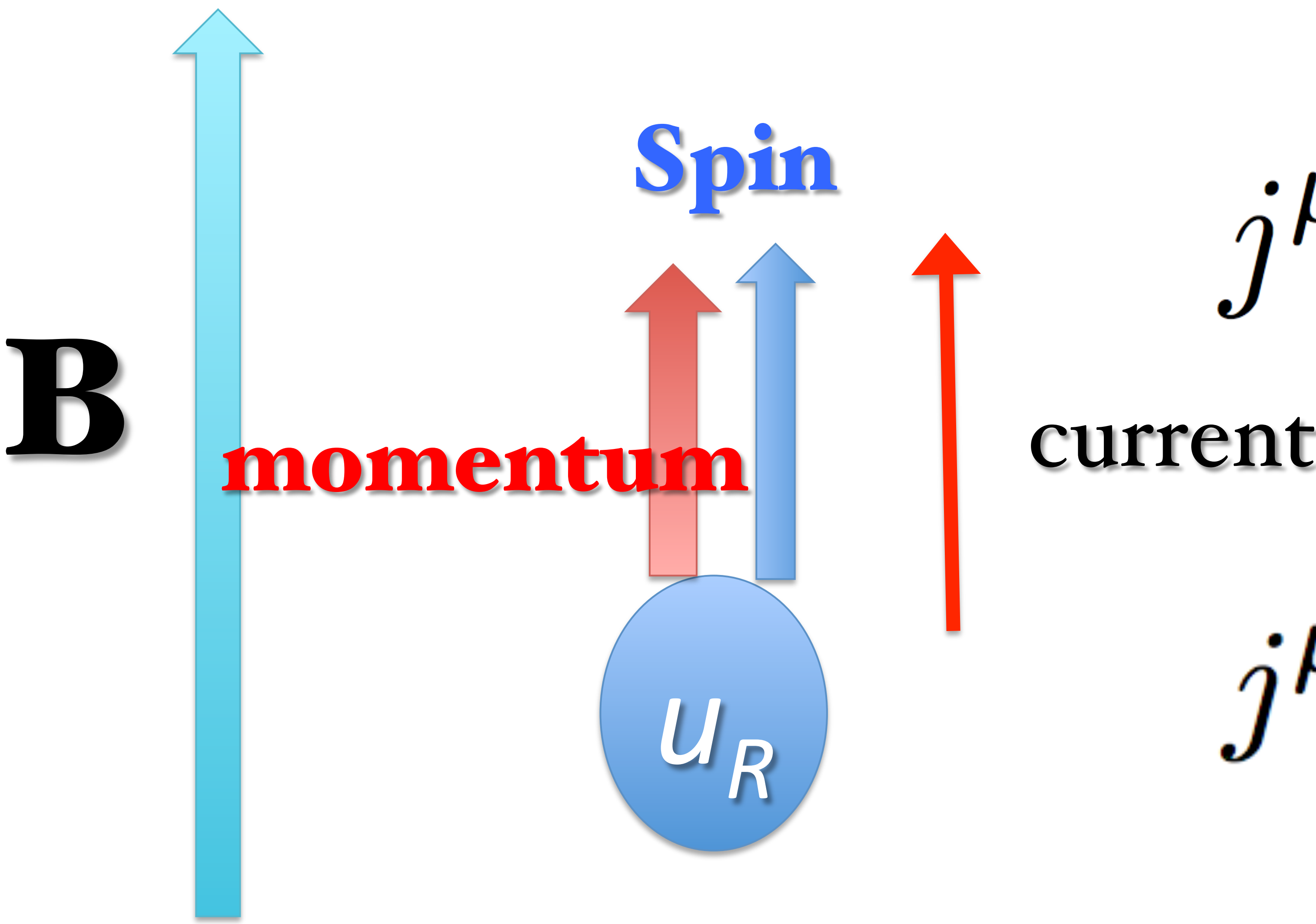


Chiral Magnetic Effect

If Number of uL ≠ Number of uR
A electric current will be observed.



Chiral Magnetic Effect (CME)

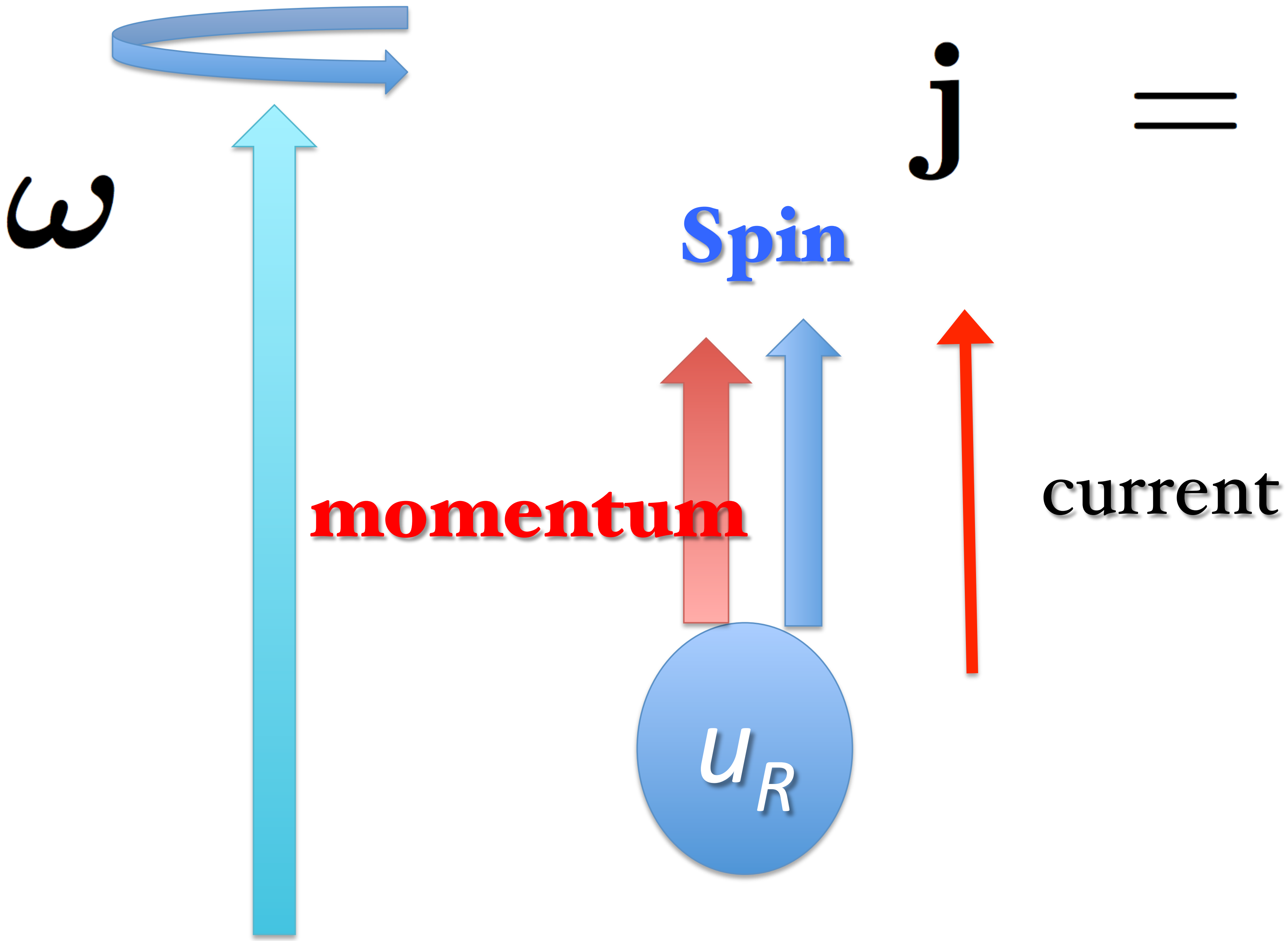


$$j^\mu = \xi_B B^\mu,$$

$$j^\mu = \sigma E^\mu,$$

Chiral Vortical Effect (CVE)

$$\mathbf{j} = \xi \omega$$



Vorticity

Vorticity, 4D covariant angular velocity

Velocity of a macroscopic object

$$\boxed{\omega^\mu} = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

$$u^\mu = (1, 0, 0, 0)$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

Chiral Magnetic and Vortical Effect

Magnetic field

Charge current

Vorticity

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$

Chiral Magnetic Effect (CME)

Chiral Vortical Effect (CVE)

Chiral Magnetic and Vortical Effect

Charge current

Magnetic field

Vorticity

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$

$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

Axial current

New Transport coefficients

$$
\begin{aligned}
j^\mu &= \xi_B B^\mu + \xi \omega^\mu, \\
j_5^\mu &= \xi_{5B} B^\mu + \xi_5 \omega^\mu,
\end{aligned}
$$

- Strong coupling, AdS/CFT duality,

(Erdmenger('09), Banerjee('11), Torabian('11), ...)

- Weakly coupling, Kubo formula

(Fukushima('08), Kharzeev('11), Landsteiner('11), Hou('12), ...)

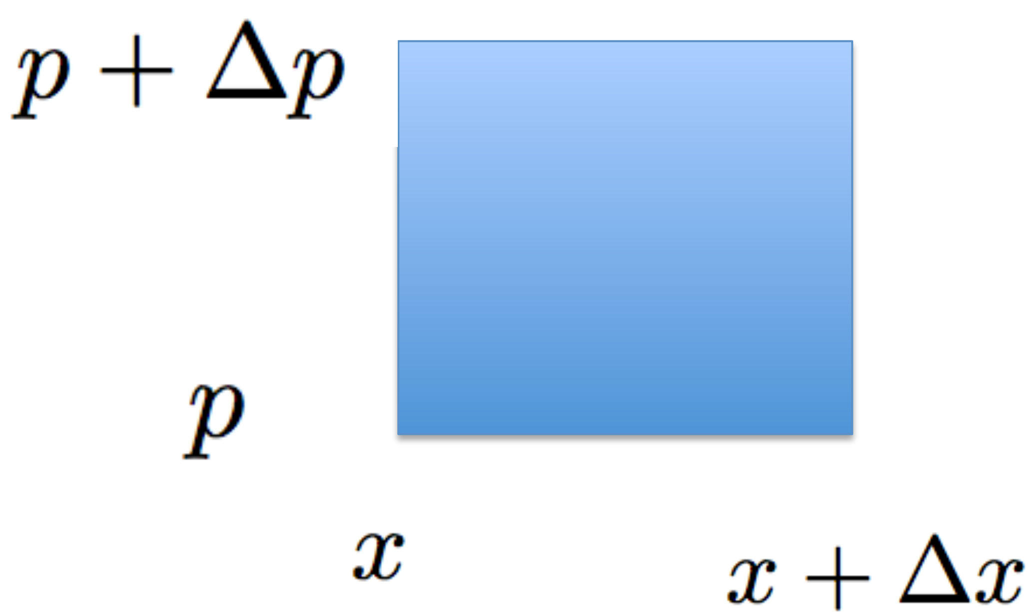
How about the kinetic theory?

- Evolution of the system
- Numerical simulations

Kinetic theory

- Kinetic theory: a microscopic dynamic theory for many-body system, to compute transport coefficients.

- distribution function, e.g. Fermi-Dirac distribution f(x,p)



Winger function for fermions

- Winger function: a quantum distribution function, ensemble average, normal ordering

Vasak, Gyulassy and Elze ('86,'87,'89)

$$W(x,p) = \langle : \int \frac{d^4y}{(2\pi)^4} e^{-ipy} \overline{\psi}(x+\frac{1}{2}y) \otimes \boxed{\mathcal{P}U(x,y)} \psi(x-\frac{1}{2}y) : \rangle$$

Gauge link

Macroscopic quantities

Charge current

$$j^\mu(x) \equiv <: \overline{\psi}(x)\gamma^\mu\psi(x) :>= \int d^4p \mathrm{Tr}\, (\gamma^\mu W),$$

Axial (chiral) current

$$j_5^\mu(x) \equiv <: \overline{\psi}(x)\gamma^5\gamma^\mu\psi(x) :>= \int d^4p \mathrm{Tr}\, (\gamma^5\gamma^\mu W),$$

Master equation from Dirac Eq.

- Massless, constant external electromagnetic fields $F^{\mu\nu}_{ext}$, turn off all internal interactions

$$[\boxed{\gamma^\mu p_\mu} + \frac{1}{2}i\hbar\gamma^\mu\boxed{(\partial^x_\mu - QF^{ext}_{\mu\nu}\partial^p_\mu)}]W = 0,$$

- First order differential equation, solve it order by order

Solve the Master equation

- Gradient expansion to Winger function W and its master equation,
  - expand all quantities at the power of derivatives
    $O(\partial_x^1), O(\partial_x^2),$
  - external fields are weak $F^{\mu\nu} \sim \partial_x^\mu A^\nu \sim O(\partial^1),$

Leading order

- 0^{th} order, non-interacting ideal gas
  - classical Fermi-Dirac distribution

- input
  - finite temperature T,
  - chemical potential $\mu = \mu_R + \mu_L$,
  - chiral chemical potential $\mu_5 = \mu_R - \mu_L$

1st order, Chiral anomaly

- Remarkable, we obtain the chiral anomaly by Winger function!

Energy momentum conservation

$$\partial_\mu T^{\mu\nu} = QF^{\nu\rho} j_\rho,$$

$$\partial_\mu j^\mu \;=\; 0,$$

Triangle anomaly

$$\partial_\mu j_5^\mu \;=\; \boxed{-\frac{Q^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}$$

$$\propto E \cdot B$$

Chiral magnetic and vortical effect

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$
$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

Consistent with other approaches!

$$\xi = \frac{1}{\pi^2}\mu\mu_5,$$
$$\xi_B = \frac{Q}{2\pi^2}\mu_5,$$
$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right),$$
$$\xi_{B5} = \frac{Q}{2\pi^2}\mu.$$

Q: charge
T: temperature

Chemical potentials

$$\mu = \mu_R + \mu_L,$$
$$\mu_5 = \mu_R - \mu_L,$$

Parity transform

$$\vec{\mathbf{j}} = \xi\vec{\omega} + \xi_B\vec{\mathbf{B}}, \rightarrow -\vec{\mathbf{j}} = (-\xi)\vec{\omega} + (-\xi_B)\vec{\mathbf{B}},$$

$$\mu_5 = \mu_R - \mu_L,$$

$$\begin{aligned} \xi &= \frac{1}{\pi^2}\mu\mu_5, \\ \xi_B &= \frac{Q}{2\pi^2}\mu_5, \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{x}} &\rightarrow -\vec{\mathbf{x}}, \\ \vec{\mathbf{j}}, \mu_5 &\rightarrow -\vec{\mathbf{j}}, -\mu_5 \\ \vec{\mathbf{B}}, \vec{\omega}, \vec{\mathbf{j}_5}, \mu &\rightarrow \vec{\mathbf{B}}, \vec{\omega}, \vec{\mathbf{j}_5}, \mu, \end{aligned}$$

Parity transform

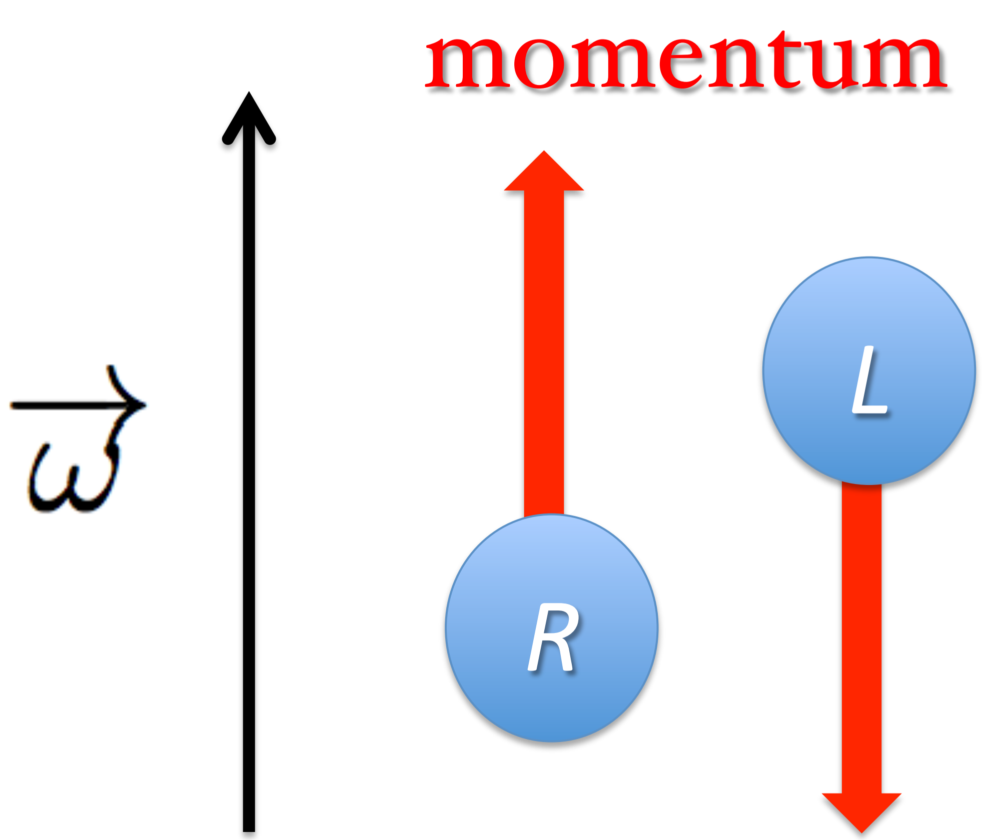
$$\vec{\mathbf{j}}_5 = \xi_5 \vec{\omega} + \xi_{5B} \vec{\mathbf{B}}, \to \vec{\mathbf{j}}_5 = (+\xi_5)\vec{\omega} + (+\xi_{5B})\vec{\mathbf{B}},$$

$$\xi_5 = \frac{1}{6}\boxed{T^2} + \frac{1}{2\pi^2}\left(\boxed{\mu^2} + \boxed{\mu_5^2}\right),$$

$$\xi_{B5} = \frac{Q}{2\pi^2}\boxed{\mu}.$$

$$\begin{aligned} \vec{\mathbf{x}} &\to -\vec{\mathbf{x}}, \\ \vec{\mathbf{j}}, \mu_5 &\to -\vec{\mathbf{j}}, -\mu_5 \\ \mathbf{B}, \vec{\omega}, \vec{\mathbf{j}}_5, \mu &\to \vec{\mathbf{B}}, \vec{\omega}, \vec{\mathbf{j}}_5, \mu, \end{aligned}$$

Prediction: Local Polarization Effect



Axial current

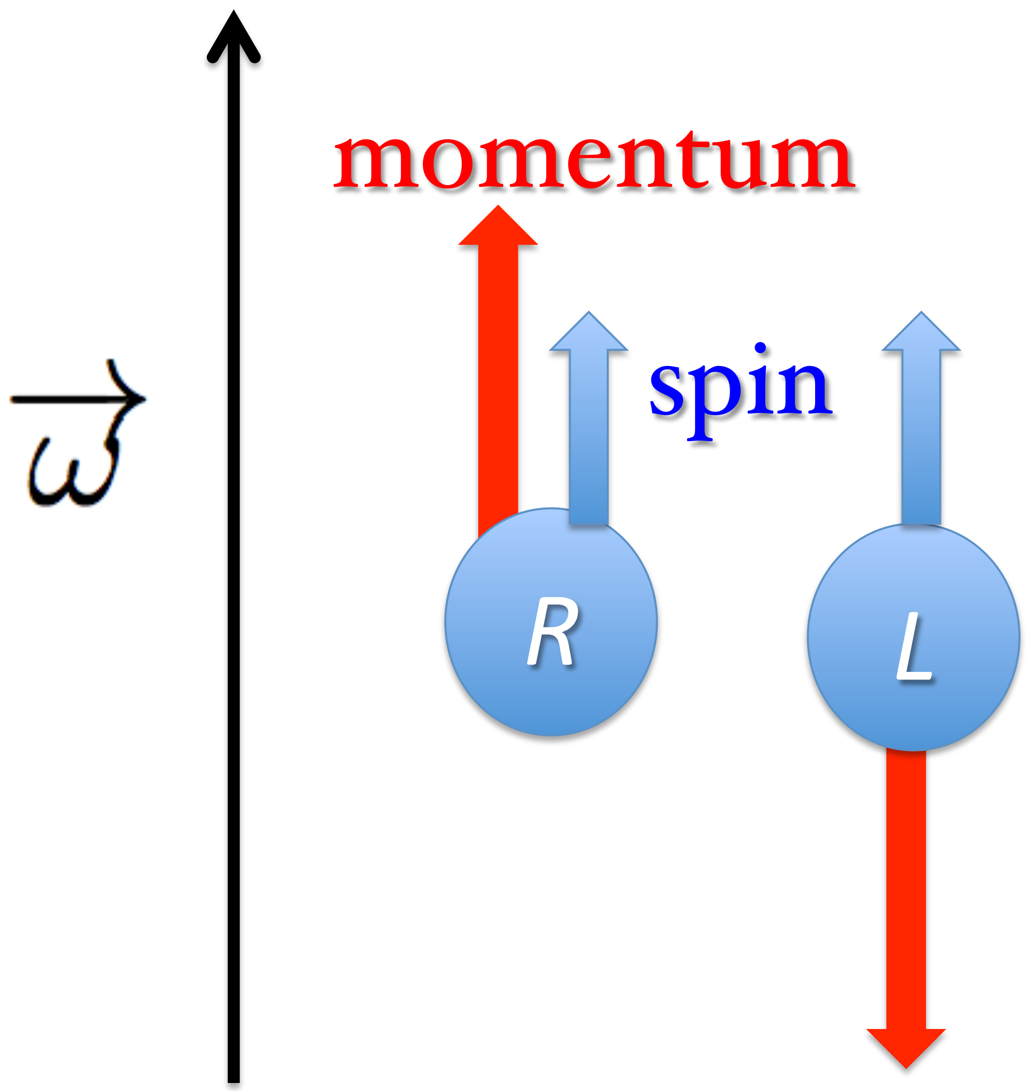
$$j_5^\mu \equiv j_R^\mu - j_L^\mu = \xi_5 \omega^\mu,$$

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2),$$

Local Polarization Effect

Spin local polarization effect

Axial current



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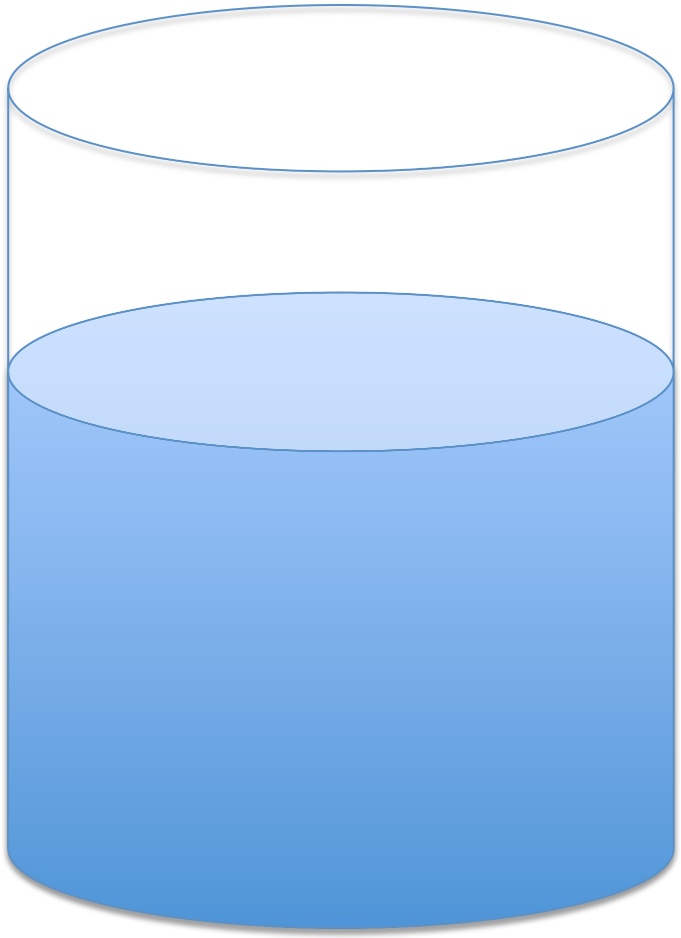
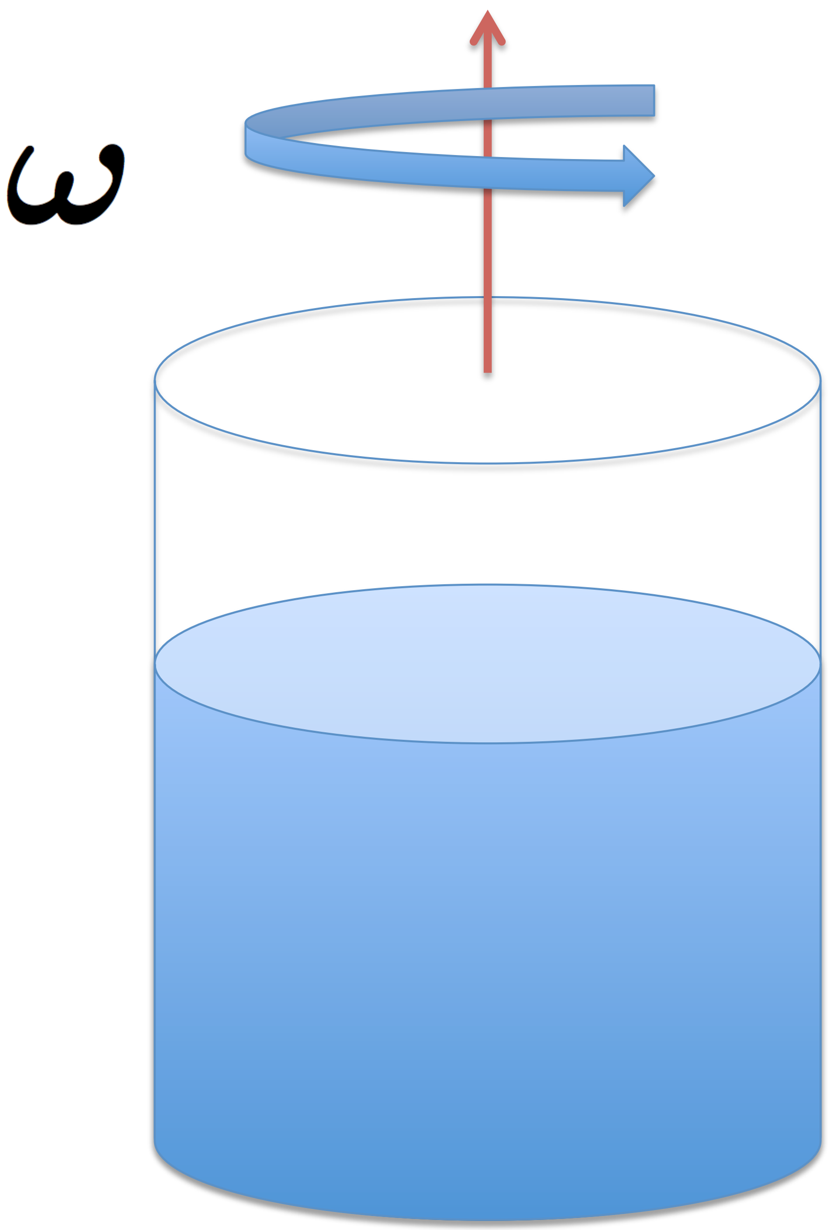
Local Polarization Effect



- a cup of water

The spins of particles are random.

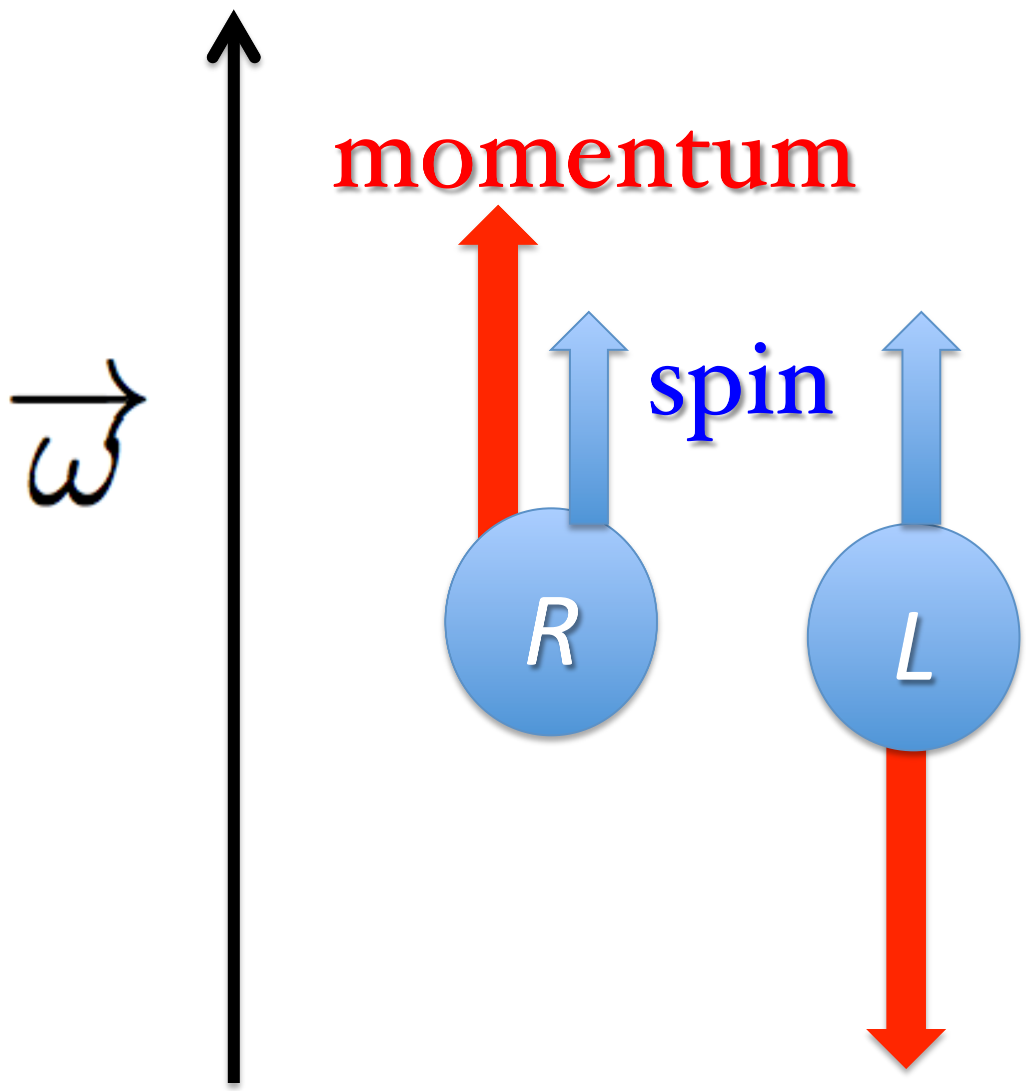
Local Polarization Effect



- rotating system,
- In that rotating frame, there will be a additional force, the Coriolis force.
- The fermions will be polarized.

Prediction (2): Local Polarization Effect

Spin local polarization effect



Axial current

$$j_5^\mu \equiv j_R^\mu - j_L^\mu = \xi_5 \omega^\mu,$$

$$\xi_5 = \frac{1}{6}\boxed{T^2} + \frac{1}{2\pi^2}\boxed{(\mu^2 + \mu_5^2)},$$

Can be observed in both high/low energy collisions

Hamiltonian approaches with Berry phase

- Son and Yamamoto, (*PRL 109, 181602*), Stephanov and Yin, (*PRL 109, 162001*), obtained the chiral anomaly, magnetic effect by Hamiltonian approaches with Berry phase.

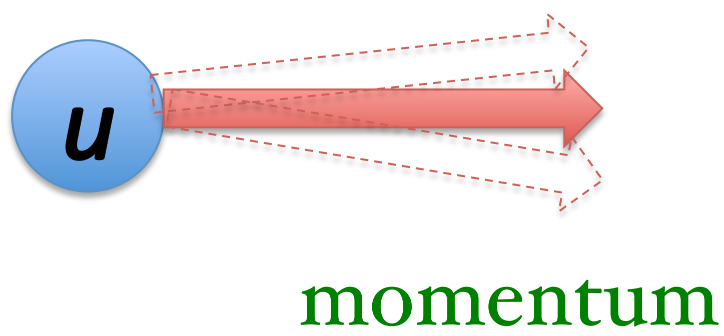
Berry phase

- Berry phase: a non-trivial phase factor of the wave function under an adiabatic process (a loop)

- related to topological phenomena in condense matter, e.g. Hall effect, spin Hall effect.

- understand chiral anomaly and chiral magnetic effect by Berry phase.

Berry phase for Dirac fermions



- Consider a free massless right-handed fermion, which has a very small fluctuations in its momentum direction.

- After a short time evolution, it goes back to the initial state, which gives a additional phase factor in momentum space.

- Just like Aharonov–Bohm phase

Berry phase

$$\exp\left[-i\int_{t_0}^{t_1} dt\,\dot{\vec{p}}\cdot\vec{a_p}\right]$$

time derivate

Berry connection

Berry curvature

- Just like a gauge field, Berry connection depends on the path in momentum space.

- One prefers the path independent quantity, Berry curvature, which is like magnetic field in U(1) gauge theory.

$$\exp\left[-i\int_V d\overrightarrow{p}\cdot\overrightarrow{a_p}\right] = \exp\left[-i\int d\overrightarrow{S_p}\cdot\overrightarrow{\Omega_p}\right]$$

$$\Omega_p = \nabla_p \times a_p,$$

Gauge field VS. Berry phase

A "gauge" field in momentum space!

Gauge theory	Berry "things"
local at x space	at p space
gauge field $\vec{A}$	Berry connection $\vec{a_p}$
magnetic field $\vec{B} = \nabla \times \vec{A}$	Berry curvature $\vec{\Omega_p} = \nabla_p \times \vec{a_p}$
Aharonov–Bohm phase $\int_V d\vec{x} \cdot \vec{A} = \iint_S d\vec{S} \cdot \vec{B}$	Berry phase $\int_V d\vec{p} \cdot \vec{a_p} = \iint_S d\vec{S_p} \cdot \vec{\Omega_p}$
Dirac monopole (magnetic charge) $\int d^3x \nabla \cdot \vec{B} = const.$	Berry monople $\int d^3p \nabla_p \cdot \Omega_p = const.$

Equation of motion of phase space

effective velocity

$$\dot{\mathbf{x}} = \frac{1}{1+\boxed{\mathbf{B}\cdot\Omega_p}}\left(\frac{\mathbf{p}}{|\mathbf{p}|}+\boxed{\mathbf{E}\times\Omega_p}+\boxed{\frac{\mathbf{p}\cdot\Omega_p}{|\mathbf{p}|}\mathbf{B}}\right),$$

$$\dot{\mathbf{p}} = \frac{1}{1+\boxed{\mathbf{B}\cdot\Omega_p}}\left(\mathbf{E}+\frac{\mathbf{p}}{|\mathbf{p}|}\times\mathbf{B}+\boxed{(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}_p}\right),$$

effective force

Berry phase will modify the velocity of paricles and give an another external force!

Q. Niu ('08)

Current

- Once the effective velocity and force are obtained, one can get all macroscopic quantities,

$$\vec{\mathbf{j}} \equiv \int \frac{d^3p}{(2\pi)^3} \dot{\vec{\mathbf{x}}} f_R,$$

lesson: chiral magnetic effect, chiral anomaly, are related to the Berry phase

velocity

distribution function for right-handed fermions

Back to Winger function

- 3-dim Hamiltonian approaches:
  - Not Lorentz covariant
  - No vortical effects

- Winger function:
  - 4-dim covariant form,
  - including the vortical effects.

Back to Wigner function

- We rewrite the master equations for the Wigner function by using the solutions and obtain a new kinetic equation for L or R quarks. We called it chiral kinetic equation.

4-dim chiral kinetic theory

4-dim Lorentz covariant evolution equation for the distribution functions

$$\boxed{\delta(p^2)}\left[\boxed{\frac{dx^\sigma}{d\tau}}\partial_\sigma^x + \boxed{\frac{dp^\sigma}{d\tau}}\partial_\sigma^p\right] f_{R/L} = 0,$$

on-shell velocity force

distribution function for Left and Right fermions

τ: proper time

Equation of motion of phase space

$$\frac{dx^\sigma}{d\tau} = p^\sigma \pm Q\left[(u\cdot b)B^\sigma - (b\cdot B)u^\sigma + \epsilon^{\sigma\alpha\beta\gamma}u_\alpha b_\beta E_\gamma\right]$$
$$\pm\left[\frac{1}{2}\omega^\sigma + \omega^\sigma(p\cdot u)(b\cdot u) - 2u^\sigma(p\cdot\omega)(b\cdot u)\right],$$
$$\frac{dp^\sigma}{d\tau} = -Qp_\rho F^{\rho\sigma} \mp Q^2(E\cdot B)b^\sigma$$
$$\pm Q\frac{1}{2}(\omega\cdot E)u^\sigma \mp Q(p\cdot\omega)b_\eta F^{\sigma\eta}\ .$$

$$b^\mu = -\frac{p^\mu}{p^2},$$

Equation motion of phase space (x,p)

Reduce to 3-dim

Integral over p0

evolution equation for distribution functions

$$\boxed{\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,}$$

$$\begin{aligned}
\frac{dt}{d\tau} &= 1 \pm Q\mathbf{\Omega}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\mathbf{\Omega}\cdot\boldsymbol{\omega}), \\
\frac{d\mathbf{x}}{d\tau} &= \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{\Omega}) \pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega}, \\
\frac{d\mathbf{p}}{d\tau} &= Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega} \\
&\quad \mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\mathbf{\Omega} \pm 3Q(\mathbf{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}},
\end{aligned}$$

equation of motion of phase space

Reduce to 3-dim

Integral over p0

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\begin{aligned}
\frac{dt}{d\tau} &= 1 \pm Q\boldsymbol{\Omega}\cdot\mathbf{B} \\
\frac{d\mathbf{x}}{d\tau} &= \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\boldsymbol{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\boldsymbol{\Omega}) \\
\frac{d\mathbf{p}}{d\tau} &= Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega}
\end{aligned}$$

if set ω=0, it is as the same as the Hamiltonian approaches.

3-dim Berry phase is embedded in our formulism!

Reduce to 3-dim

Integral over p0

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\begin{aligned}
\frac{dt}{d\tau} &= 1 \pm Q\boldsymbol{\Omega}\cdot\mathbf{B} \boxed{\pm 4|\mathbf{p}|(\boldsymbol{\Omega}\cdot\boldsymbol{\omega})}, \\
\frac{d\mathbf{x}}{d\tau} &= \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\boldsymbol{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\boldsymbol{\Omega}) \boxed{\pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega}}, \\
\frac{d\mathbf{p}}{d\tau} &= Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega} \\
&\quad \boxed{\mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\boldsymbol{\Omega} \pm 3Q(\boldsymbol{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}}},
\end{aligned}$$

ω dependence

is new!

4-dim currents

Compared to 3-dim form

$$\vec{\mathbf{j}} \equiv \int \frac{d^3p}{(2\pi)^3} \dot{\vec{\mathbf{x}}} f_R,$$

Define

$$j^{\sigma}_{R/L} = \int d^4p\,\delta(p^2) \frac{dx^{\sigma}}{d\tau} f_{R/L}$$

It gives chiral magnetic and vortical effects!

4-dim Euclidean Monoples

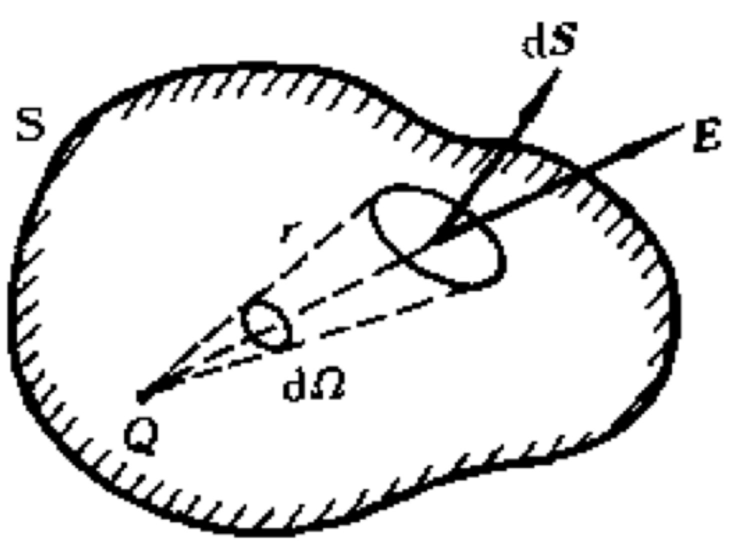
- We find those chiral effects and anomaly are related to the following term,

$$b^\mu \delta(p^2) = -\frac{p^\mu}{p^2}\delta(p^2),$$

- divergent
- play a role as a 4-dim delta function

4-dim Euclidean Monopoles

$$\boxed{\int d^4p\,\partial^p_\sigma[b^\sigma\delta(p^2)] = \frac{1}{\pi}\int d^4p_E\,\partial^{p_E}_\sigma\left(\frac{p_E^\sigma}{p_E^4}\right) = 2\pi}$$



- analogy to the volume integration of divergence of electric field.

Gauss theorem $\int dV\nabla\cdot\vec{E} \propto Q,$

There is a source in the momentum space!

4-dim Euclidean Monopoles

$$\int d^4p\partial_\sigma^p[b^\sigma\delta(p^2)] = \frac{1}{\pi}\int d^4p_E\partial_\sigma^{p_E}\left(\frac{p_E^\sigma}{p_E^4}\right) = 2\pi$$

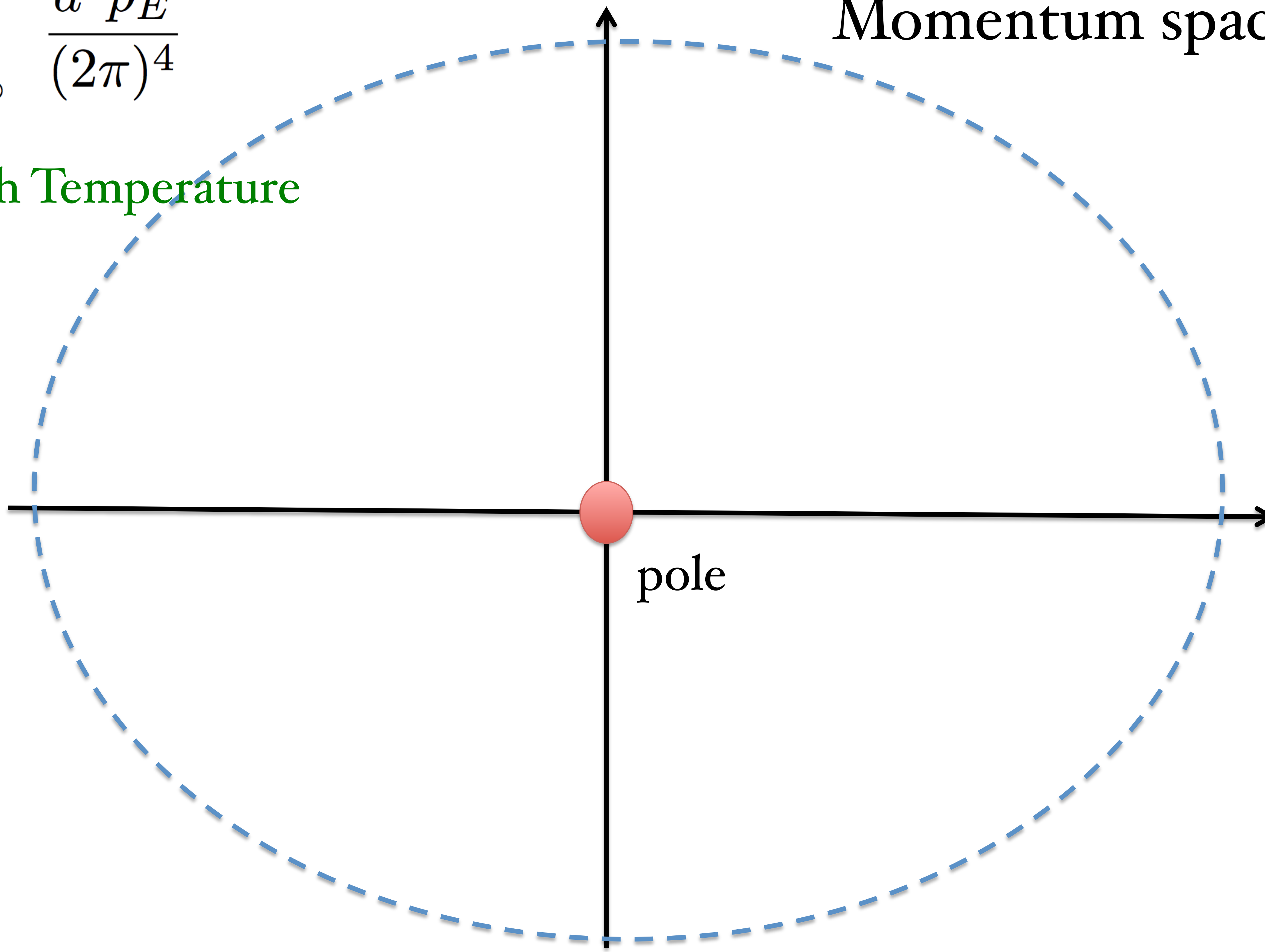
- There is a source in the momentum space!

- It is from the fact you cannot find a on-shell massless particle with a zero momentum. So the fermion sphere has a hole at the zero point (Euclidean).

$$\int_{-\infty}^{+\infty} \frac{d^4 p_E}{(2\pi)^4}$$

Momentum space

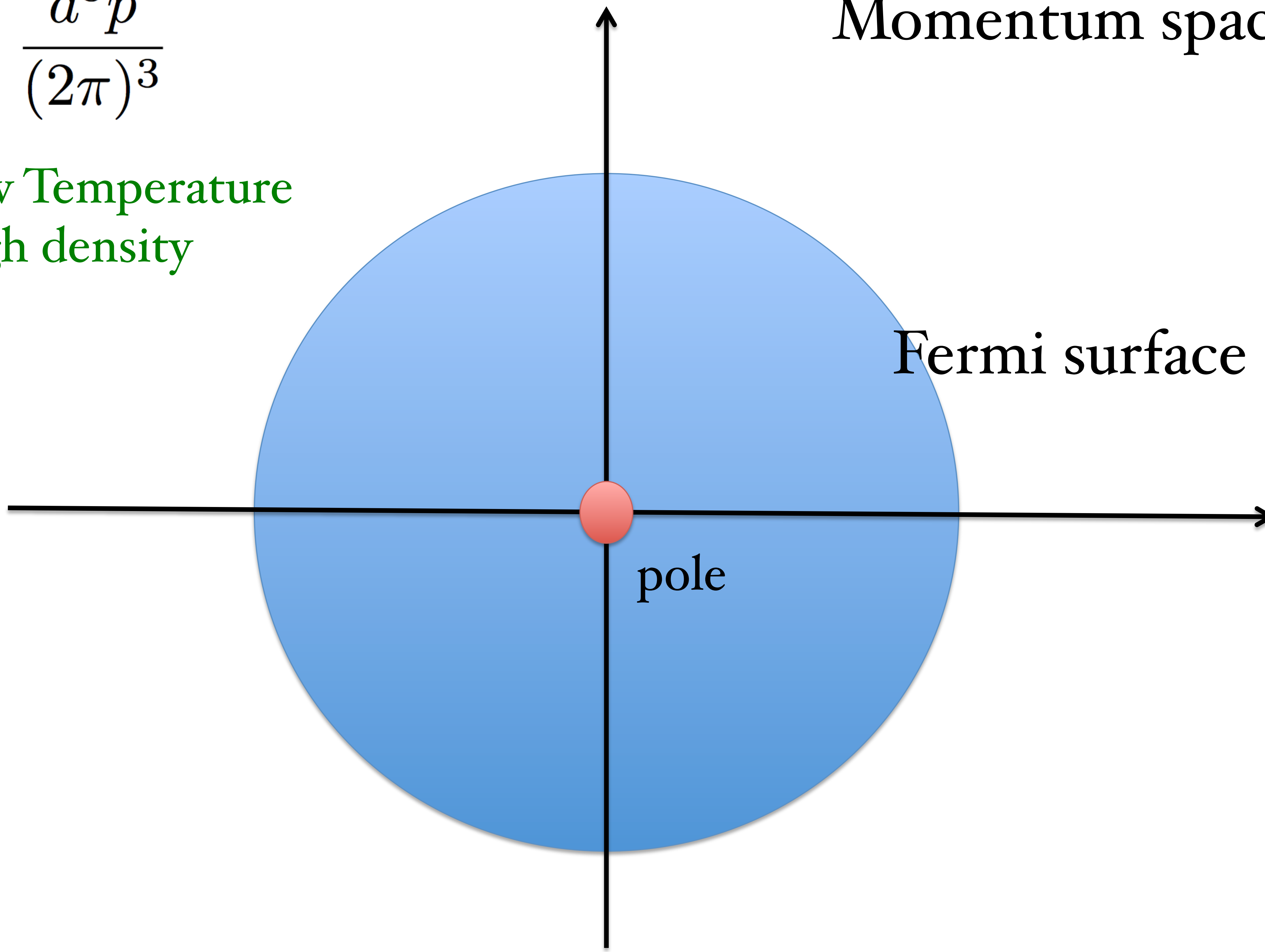
High Temperature case



$$\int_0^{\mu} \frac{d^3p}{(2\pi)^3}$$

Low Temperature
High density

Momentum space



- Whatever in high temperature or high density case, the pole will give a source and finally contribute to the axial current and gives the chiral anomaly.

- That is why our approach (high temperature) is found to be equivalent to the others (e.g. Fermi-liquid).

Summary

- We use Winger function to obtain
  - Chiral magnetic (CME) and vortical effect (CVE), chiral anomaly are induced automatically.
  - The spin local polarization effect can be observed in high/low energy collisions.

Summary

- We get a 4D Lorentz covariant chiral kinetic theory with Berry phase.
  - This provides a unified interpretation of the chiral magnetic and vortical effects, chiral anomaly, and Berry phase in the framework of Wigner functions.
  - We find the coupling between Berry phase and vorticity (dynamic quantity).

Thank you!